Technical Comments

Comment on "Minimum Control Power for VTOL Aircraft Stability Augmentation"

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PROFESSOR Swaim continues to propagate an error in writing the equations of motion for a hovering VTOL aircraft in a uniform wind.[1] This improper formulation can cause a noticeable difference in the calculation of the dynamic motions of a vehicle.[2]

The error contained in the analysis has been pointed out in a Note by Smith,[3] but has not been corrected,[1] and therefore further discussion seems desirable.

The equations of motion of an aircraft are written by an observer traveling at a chosen uniform translational velocity with respect to the vehicle. An Eulerian or body axis system is normally used. U_0 is defined as the velocity of the axis system in the direction of the X axis and is selected to coincide with the trim velocity of the aircraft measured by the observer. The observer normally selects a frame of reference such that the air mass is at rest. U_0 is then the velocity of the aircraft with respect to the air mass. The observer may, of course, select any other translational velocity at his convenience.

Consider the example problem of a VTOL aircraft hovering in a uniform wind of magnitude U_w.[1] If the observer chooses a reference frame translating with the velocity of the wind, then the trim velocity of the aircraft measured by the observer is U_w, and the observer sees the "conventional" situation of a vehicle traveling at the velocity U_w into still air and U_0 is equal to U_w. If the observer chooses a reference frame such that the aircraft is stationary in the trim state, then the trim velocity of the aircraft U_0 is equal to zero and the air mass is translating at the velocity U_w.

There is a difference in the inertial terms in the equations of motion in each of these reference frames owing to the difference in U_0. There is also a difference in the aerodynamic variables, which must be expressed as a function of the variables describing the motion of the axis system in each of these reference frames. In particular, as pointed out by Smith,[3] the lateral velocity with respect to aircraft body axes is calculated differently. If the observer chooses the frame in which the aircraft is stationary, the wind velocity U_w does not rotate with the axis system. The aerodynamic forces and moments depend upon the relative wind velocity in the direction of the lateral body axis of the aircraft. This velocity is equal to $(v\text{-}U_w\psi)$, where v is the velocity of the Eulerian axis in the Y direction and ψ is the yaw angle, which locates the X axis with respect to the wind direction. If the aerodynamic variables are properly formulated in terms of axis motion variables, the equations of motion are identical in either reference frame. Both observers will calculate identical dynamic characteristics for the vehicle.

The dynamic motions of a vehicle must be invariant with changes in the translational velocity of an observer and depend only on the velocity of the air with respect to the vehicle. If this were not the case, then it is implied that a device could be constructed which would determine the ground speed of an aircraft without reference to the ground. Velocities are, of course, only defined with respect to an observer. An observer situated in an airplane with no ground reference has no means by which to determine the presence of a uniform wind.

References

[1] Swaim, R. L., "Minimum Control Power for VTOL Aircraft Stability Augmentation," *Journal of Aircraft*, Vol. 7, No. 3, May–June 1970, pp. 231–235.

[2] Swaim, R. L., "VTOL Aircraft Dynamics," *Journal of Aircraft*, Vol. 6, No. 2, March–April 1969, p. 176.

[3] Smith, R. H., "Reply by the Author to R. L. Swaim," *Journal of Aircraft*, Vol. 6, No. 2, March–April 1969, p. 176.

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Reply by Author to H. C. Curtiss Jr.

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CURTISS and Smith are correct. One of the assumptions nearly always made in deriving equations of motion for conventionally flying aircraft is that the atmosphere rotates with the Earth and the observer reference frame is an inertial axis system fixed on the Earth. This assumption is not correct when a mean wind (relative motion between the atmosphere and Earth) is present. Then, as Curtiss points out, the observer's frame translates with the mean wind if one wishes to use the equations of motion as usually formulated in a body axis system.

The stability augmentation system design method put forth in Ref. 1 is still valid, only some of the numbers in the numerical example illustrating the method are changed because of the incorrect omission of the $U_0\psi$ term in the sideslip equation. Carrying through the example with the term included results in Eq. (37) becoming $CP_{\text{available}} = 0.0169\ \sigma_{v_g}$. For a 14 fps rms value of σ_{v_g}, the yaw control power is 0.237 rad/sec² instead of 1.168 as given in Ref. 1.

Reference

[1] Swaim, R. L., "Minimum Control Power for VTOL Aircraft Stability Augmentation," *Journal of Aircraft*, Vol. 7, No. 3, May–June 1970, pp. 231–235.

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